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THEORETICAL OF DYNAMIC LOADING AGAINST WATER-FILLED SIMPLY SUPPORTED PIPES

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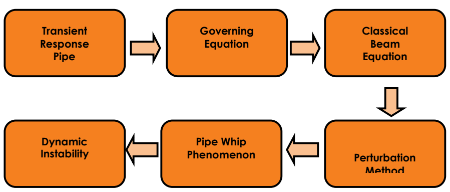
Graphical abstract



Abstract

The primary aim of this study had been to investigate the effects of water-filled flow on the transient response of a simply supported pipe subjected to dynamically applied loading. The importance of this study is manifested in numerous applications, such as oil and gas transportations, where dynamic loading can be the result of an accident. The classical Bernoulli-Euler beam theory was adopted to describe the dynamic behavior of an elastic pipe and a new governing equation of a long pipe transporting gas or liquid was derived. This governing equation incorporated the effects of inertia, centrifugal, and Coriolis forces due to the flowing water. This equation can be normalized to demonstrate that only two non-dimensional parameters governed the static and the dynamic responses of the system incorporating a pipe and flowing water. The transient response of this system was investigated based on a standard perturbation approach. Moreover, it had been demonstrated that the previous dynamic models, which largely ignored the internal flow effects and interactions between the flow and the structure, normally produced a large error and are inapplicable to the analysis of many practical situations. One interesting effect identified was that at certain flow ratio, the system became dynamically unstable and any, even very small, external perturbation led to a growing unstable dynamic behavior. Such behavior, which is called pipe whip, is well-known to everyone who waters a garden using a flexible long hose.

Keywords: Flowing water; pipe whip; simply supported pipe; transporting gas or liquid

Abstrak

Tujuan utama kajian ini adalah untuk menyiasat kesan respons transien terhadap aliran air di dalam paip sokongan mudah yang dikenakan beban secara dinamik. Kepentingan kajian ini dapat dilihat dalam pelbagai applikasi seperti pengangkutan minyak dan gas di mana pembebanan dinamik boleh menyebabkan terjadinya kemalangan. Teori klasik Euler-Bernoulli digunakan untuk menggambarkan tingkah laku dinamik paip elastik dan persamaan baru diterbitkan untuk paip panjang yang mengalirkan gas atau cecair. Persamaan yang diterbitkan ini menggabungkan daya inersia, daya empar, dan daya Coriolis untuk cecair di dalam paip yang mengalir. Persamaan ini dipermudahkan dengan menggunakan hanya dua parameter tanpa dimensi yang mengawal respon statik dan dinamik sistem yang menggabungkan paip dan air yang mengalir. Satu kesan menarik yang dikenal pasti dalam kajian ini adalah bahawa pada nisbah aliran tertentu, sistem menjadi tidak stabil secara dinamik dan walaupun nilai ini sangat kecil, pada nisbah yang besar, gangguan luaran ini membawa kepada perubahan tingkah laku dinamik dan mempengaruhi kestabilan sistem. Tingkah laku ini, yang dikenali sebagai *pipe whip,* adalah fenomena biasa yang sering terjadi dan keadaan ini paling dapat dilihat apabila menyiram air di taman bunga dengan menggunakan hos panjang yang fleksibel.

Kata Kunci: Aliran air; cambuk paip; paip sokongan mudah; gas atau cecair pengangkut

1.0 INTRODUCTION

Long flexible pipes are widely used across many industries, such as power generation, oil and gas, as well as petrochemical industries to transport liquids and gas, or their mixtures. In the offshore industry, pipelines are a crucial part for the distribution of oil and gas. The failure in a pipeline for transporting liquid or gas could have very great economic and environmental consequences [1,2]. In the case of an accident, these pipes are subjected to dynamic or impulse loading. Besides, pipe impact problems have been studied with a lot of different approaches. Micro-pipes carrying liquid or gas have been gaining popularity in various sensor technologies, which involve some dynamic measurements [2,3]. Therefore, there is now a growing interest in studying the transient response of pipes with flowing media caused by dynamic loading [4-6].

The nature of internal flow associated with pipe interactions is extremely complex [7,8]. The flowing medium can produce substantial forces on the pipe wall and affect the dynamic response of the structure, and, at the same time, the pipe dynamics can change the flow characteristics in a very complicated manner. Due to these highly complicated mechanisms, it is not surprising that in many previous theoretical works, investigating impact loading of pipes and internal flow effects, were just ignored or only partially incorporated into the structural response using some oversimplified theoretical assumptions [9,10]. Moreover, some studies focused on the impact against fully clamped pipes [11,12] and with the internal pressure to the pipe during impact [13,14]. The major finding was a difference in deformation between empty and pressurized pipes after impact [15]. In a number of experimental and theoretical papers, it was demonstrated that the structure-filled-medium interactions significantly influenced the dynamic behavior [16-19].

The flowing medium can lead to principally new phenomena, such as pipe instability in the absence of the applied axial loading or pipe whip behavior. The latter has the same origin as the chaos of an unrestrained garden hose with water running through it. In the case of a full-bore failure of high-pressure above ground pipeline, the pipe whip phenomenon has potential for considerable damage to any property in the vicinity and could also potentially destroy a substantial length of pipeline [7-11]. Most of the previously published works dealt with the situation when the filled medium is stationary and not many studies have addressed the pipe-flowing-medium interactions. Moreover, only a number of studies have been examining the impact against liquid-filled pipes. Furthermore, it had been found that pipe perforation occurred when water was present in the pipe during impact [20], and in the present study, an analytical model for an impulse load against a pipe with a flowing medium was proposed [21].

Another approach for estimating loads against pipelines due to trawl gear interaction [22] presented impact tests against pipes made from X65 grade offshore steel, and numerical simulations. The effect of adding content (water) to the pipe was examined [23], with both open and closed ends. The results of the experiments were generally well-reproduced [24] and the key phenomena were captured via computer simulations. Therefore, the primary purpose of this work was to develop analytical and numerical tools needed to facilitate the study of pipe dynamic behavior.

On top of that, this paper focused on the investigation of the transient response of a simply supported pipe with flowing medium to an impulse loading. Due to the approximate nature of the mathematical modelling, a number of assumptions and simplifications were introduced. To model the transient response, the classic Bernoulli-Euler beam theory was applied and this theory was extended to incorporate the flowing medium effects: inertia of the flow, centrifugal force due to the pipe curvature, as well as the Coriolis force due to the flow rotation. The derived governing equation represents a fourth order non-homogeneous partial differential equation (PDE). A dimensionless form of the governing equation contains only two dimensionless parameters controlling the system mechanical behavior. In the case of a small influence of the flowing medium on the mechanical behavior, the governing equation was solved using an approach based on the standard perturbation method.

The equations and the approaches were extended to other types of pipe constraints, as well as dynamic loading conditions. One interesting observation was the existence of critical values of the governing parameters at which any small disturbance led to growing unbound deflections of the pipe. Such behavior had been linked to the dynamic instability of pipes with flowing water, the behavior which is familiar to everyone who has experienced the chaos caused by an unrestrained hose when watering the garden. This paper is organized as follows: the theoretical modelling for the actual beam model incorporating the flowing water as specified in Section 2. Pipe whip phenomenon is discussed in Section 3. Section 4 discusses the results of the effect of the flowing water. This is followed by brief conclusions in Section 5.

2.0 GOVERNING EQUATION

In this section, the simplified governing dynamic equation of a simply supported pipe with flowing gas or liquid had been formulated [11,12]. A pipe element of length $d\bar{x}$ was considered, as shown in Figure 1.

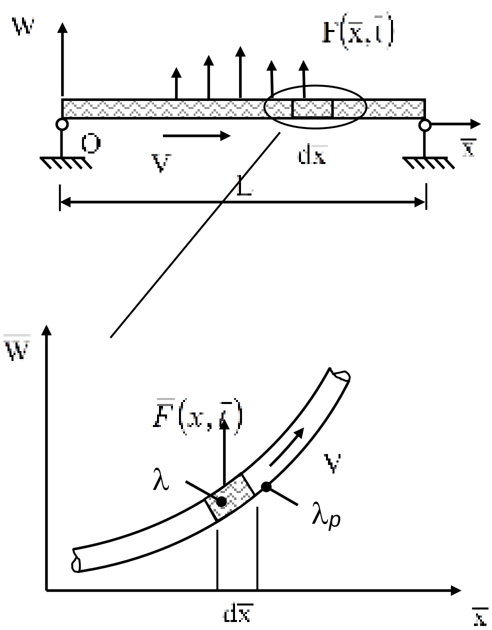


Figure 1: Dynamic equation of the system pipe-flowing liquid.

Let λ_f and λ_p be the medium and the pipe mass per unit length. In the case of the pipeline's transverse motion, the element will be subjected to the inertia force of intensity as:

$$-\frac{\partial^2\bar{W}}{\partial\bar{t}^2}\left(\lambda_f+\lambda_p\right) \tag{1}$$

Because the flow rotates with angular speed, $\frac{\partial^2\bar{W}}{\partial\bar{x}\partial\bar{t}}$, the Coriolis force acting on the element t $d\bar{x}$ will be given by:

$$-2\frac{\partial^2\bar{W}}{\partial\bar{x}\partial\bar{t}}\lambda_f V d\bar{x} \tag{2}$$

where V is the velocity of the medium. With the same sign, one can write the centrifugal force acting on the element due to the curvature of the gas flow as:

$$-\frac{\partial^2\bar{W}}{\partial\bar{x}^2}\lambda_f V^2 d\bar{x} \tag{3}$$

It is assumed for simplicity that the dynamics of the pipe do not affect significantly the flow characteristics, such as velocity and density. Thus, the structural and the fluid mechanics components of the problem were decoupled. Taking also into account had been the elastic response of the pipe element $d\bar{x}$, whereby the governing dynamic equation can be written in the following form:

$$EI\frac{\partial^4\bar{W}}{\partial\bar{x}^4}+\frac{\partial^2\bar{W}}{\partial\bar{t}^2}\left(\lambda_f+\lambda_p\right)+2\frac{\partial^2\bar{W}}{\partial\bar{x}\partial\bar{t}}\lambda_f V+\frac{\partial^2\bar{W}}{\partial\bar{x}^2}\lambda_f V^2=\bar{F}(\bar{x},\bar{t}) \tag{4}$$

where E represents Young's modulus, I is the second moment of inertia of the pipe. The right-hand side term $\bar{F}(\bar{x},\bar{t})$ represents a driving force per unit length. Besides, it was assumed that $0\le\bar{x}\le L$ and $\bar{t}\ge 0$. It is always advantageous (and often necessary) to rewrite a governing equation in a dimensionless form. To do so, the following scaling transformations were introduced:

$$x=\frac{\bar{x}}{L}, \qquad t=\frac{\bar{t}}{\tau}, \qquad W(x,t)=\frac{\bar{W}(x,t)}{L} \tag{5}$$

where τ is a parameter, which will be defined later in this paper. Then,

$$\frac{EI}{L^3}\frac{\partial^4 W}{\partial x^4}+\frac{(\lambda_f+\lambda_p)L}{\tau^2}\frac{\partial^2 W}{\partial t^2}+2\frac{V\lambda_f}{\tau}\frac{\partial^2 W}{\partial x\partial t}+\frac{V^2\lambda_f}{L}\frac{\partial^2 W}{\partial x^2}=\bar{F}(x,t) \tag{6}$$

It is straightforward to show the dimensional consistency of the equation. To proceed with the solution of the governing equation, it is convenient to recast this equation into a more compact form. Dividing both sides by EI/L^3, one would obtain:

$$\frac{\partial^4 W}{\partial x^4}+\frac{(\lambda_f+\lambda_p)L^4}{EI\tau^2}\frac{\partial^2 W}{\partial t^2}+2\frac{V\lambda_f L^3}{EI\tau}\frac{\partial^2 W}{\partial x\partial t}+\frac{V^2\lambda_f L^2}{EI}\frac{\partial^2 W}{\partial x^2}=F(x,t) \tag{7}$$

Now, one could define:

$$\tau=L^2\sqrt{\frac{\lambda_p+\lambda_f}{EI}} \tag{8a}$$

$$\varepsilon=\frac{VL\lambda_f}{\sqrt{EI(\lambda_p+\lambda_f)}} \tag{8b}$$

and

$$\beta=VL\sqrt{\frac{\lambda_p+\lambda_f}{EI}} \tag{8c}$$

$$F(x,t)=\frac{L^3}{EI}\bar{F}(x,t) \tag{8d}$$

Then, Eq (6) becomes

$$W_{,xxxx}+W_{,tt}+\varepsilon\left(2W_{,xt}+\beta W_{,xx}\right)=F(x,t) \tag{9}$$

In Eq (10), introduced the following notations for derivative $\partial W/\partial x\equiv W_{,x}$ had been introduced. A similar rule was applied to the variable t, as well as for higher-order derivatives. Observe also that following the scaling transformation in (8) and (9), the space variable x varied between 0 and 1.

In the derivation of the governing equation (9), the effect of axial forces on the transverse movements was omitted, which could develop in the pipe as a result of the viscosity of the flow or reactions of supports. Such forces can also contribute to the dynamic response at large values of pipe deflections or slopes of the deflection curve. However, in the following analysis, the deflections were assumed as small and the effect of the axial forces (due to flow viscosity and support reactions) on the transverse movements were negligible in comparison with other forces. Besides, it is important to stress that within the developed model, only two parameters, ε and β fully controlled the transient response of the system. Therefore, the derived governing equation (9) can be utilized to investigate various dynamic phenomena using reduced size or scale physical models. Such scale models would be adequate if the values of the governing parameters ε and β are kept the same for the scale model and the reference system. This can potentially result in substantial benefits if the experimental approach is adopted for the investigation of the problem under consideration. Finally, note that for this analysis, the initial and the boundary conditions were imposed to correspond to a simply supported pipe, which can be written as:

$$W(0,t) = W_{,xx}(0,t) = W(1,t) = W_{,xx}(1,t) = 0 \qquad (10)$$

Next, a solution of (9) is explained for an external force $\bar{F}(x,t)$, which is subsequently specialized to a short-duration temporal impulse.

2.1 Non-Homogenous Classical Beam Equation

The transient analysis was begun with the classical non-homogeneous beam equation subjected to the boundary conditions (10) at $x = 0$ and $x = 1$:

$$W_{,xxxx} + W_{,tt} = F(x,t) \qquad (11)$$

The driving load distribution was also assumed as $\bar{F}(x,t) = 0$ at $t < 0$. Several solution techniques can be applied to analyse the beam's transient response to an impulse perturbation, including Laplace-Laplace transforms, the method of undetermined coefficients, variation of parameters, and the Joint transform scheme [12,13]. An analytical solution of Eq (11) with boundary conditions (10) had been obtained by the separation of variables technique, while the Laplace transform [12] can be written as:

$$W(x,t) = \sum_{k=1}^{\infty} Y_k(x) \int_0^1 Y_k(u) \int_0^t F(u,\tau) \frac{\sin(\omega_k(t-\tau))}{\omega_k} d\tau du \qquad (12)$$

where Y_k are the set of normalized eigenfunctions corresponding to the solution of the homogeneous equation $W_{,xxxx} + W_{,tt} = 0$, subjected to the specified boundary conditions:

$$Y_k(x) = \sqrt{2}\, sin\, \lambda_k x, \qquad \lambda_k = k\pi \qquad (13)$$

and

$$\omega_k = \lambda^2{}_k = (k\pi)^2 \qquad (14)$$

The coefficient $\sqrt{2}$ in eigenfunctions, Eq (13), is a normalization constant that ensures that the eigenfunctions are not only orthogonal, but also orthonormal. Eq (12) provides the general analytical expression for the transient spatio-temporal dynamics of a simply supported beam driven by arbitrary load distribution $F(x,t)$. As an example, consider a spatio-temporal impulse driving force at time $t = 0$ with a magnitude f_a (in scaled variables). Hence, Dirac's time-and-space delta function was used to model this impulse. Then, $F(x,t)$ takes the form

$$F(x,t) = f_a \cdot \delta(x^*)\delta(t) \qquad (15)$$

where x^* represents the position along the pipe at which the driving impulse is applied.
Carrying out the space and time integrations, as well as invoking the basic properties of Dirac's delta function, one could obtain:

$$W(x,t) = 2f_a \sum_{k=1}^{\infty} \sin(\lambda_k x) \sin(\lambda_k x^*) \frac{\sin(\omega_k(t-\tau))}{\omega_k} \qquad (16)$$

The above equation is well-known and describes the mechanical response of a simply supported beam subjected to a spatio-temporal impulse and this equation represents the exact analytical solution to problem [10].

2.2 Perturbation Method

Next is the case of a simply supported pipe with flowing water. In this case, the simplified governing equation was derived in the previous section and can be written as:

$$W_{,xxxx} + W_{,tt} + \varepsilon\left(2W_{,xt} + \beta W_{,xx}\right) = F(x,t) \qquad (9)$$

where two parameters, ε and β fully control the transient response of the system. As a result, the derived governing equation (9) was used to investigate various dynamic phenomena using a reduced size physical model. Such scale modelling is appropriate if the values of ε and β are kept the same for the scale model and reference conditions. This can potentially result into substantial benefits if an experimental approach is adopted for the investigation.

At fixed β, Eq (9) was reduced to the classic Bernoulli-Euler equation when both products ε and β were limited to zero. Furthermore, the smallness of this parameter was exploited in the following perturbation analysis. In contrast to the previously considered case, an attempt at a standard separation of variables approach to determine the dynamic response failed because it was impossible to satisfy the boundary conditions with a non-trivial solution. However, the problem can be solved as a regular perturbation problem with ε as a small parameter, that is, the solution has the form

$$W = W_0 + \varepsilon W_1 + \varepsilon^2 W_2 + \cdots \varepsilon^m W_m + \cdots \tag{17}$$

If the expansions (17) are substituted into Eq (9) and like powers of ε are equated, then the following expressions for the deflection W, correct up to order ε^m, are obtained,

$$\left.\begin{array}{c} W_{0,xxxx} + W_{0,tt} = F(x,t) \\ W_{1,xxxx} + W_{1,tt} = F_0(x,t) \\ F_0(x,t) = -2W_{0,xt} - \beta W_{0,xx} \\ \\ W_{2,xxxx} + W_{2,tt} = F_1(x,t) \\ F_1(x,t) = -2W_{1,xt} - \beta W_{1,xx} \end{array}\right\}$$

$$\cdots$$
$$W_{m,xxxx} + W_{m,tt} = F_{m-1}(x,t)$$
$$F_{m-1}(x,t) = -2W_{m-1,xt} - \beta W_{m-1,xx} \tag{18}$$

These equations are subjected to the specified boundary conditions (10). The above equations can be considered as a recurrent system that can be solved through a step-by-step integration using the general solution for the simply supported beam subjected to arbitrary load distribution (12), which can be written as:

$$W_m(x,t) = 2\sum_{k=1}^{\infty} \sin(\lambda_k x) \int_0^1 \sin(\lambda_n u) \int_0^t F_{m-1}(u,\tau) \frac{\sin(\omega_k(t-\tau))}{\omega_k} d\tau du \tag{19}$$

The above recurrent equation (18) and the general solution for non-homogeneous beam, equation (19), provide an analytical tool for the analysis of the effect of the flowing media on the transient response of pipes.

3.0 PIPE WHIP PHENOMENON

In this section, the conditions corresponding to the initiation of the dynamic instability of the pipe had been investigated. For the convenience of the reader, the homogeneous governing equation of a pipe had been rewritten below with the flowing medium:

$$W_{,xxxx} + W_{,tt} + \varepsilon\left(2W_{,xt} + \beta W_{,xx}\right) = 0 \tag{9}$$

Furthermore, assuming the solution in the form

$$W(x,t) = w(x)e^{(\eta + i\omega)t} \tag{20}$$

then the governing equation (9) can be rewritten as:

$$w_{,xxxx} + (\eta + i\omega)^2 w + \varepsilon\left(2(\eta + i\omega)w_{,x} + \beta w_{,xx}\right) = 0 \tag{21}$$

Mathematically, the dynamic instability takes place when the real part of exponent index $\eta + i\omega$ of the assumed solution (21) takes positive values. In this case, theoretically, the amplitude of oscillations can grow unbound with time. However, in practice, the amplitude of oscillations is limited and controlled by the damping properties of the pipeline structure, which always exist in real conditions. Neglecting the damping, thus, the problem is reduced to determination of the domain of parameters ε and β variation, at which $\eta > 0$. Thus, at fixed ε and $\eta = 0$, one would need to determine minimum β, which satisfies Equation (22) with an additional condition $\partial\eta/\beta > 0$ that corresponds to the transition from stable conditions to dynamic instability characterised by the growing amplitude of oscillations with time.

Following [12, 13], the solution in the interval $0 \le x \le 1$ is represented in the following form:

$$w(x) = \sum_{n=0}^{\infty} C_n x^n \tag{22}$$

The boundary conditions at the left end (fixed end) follows that $C_0 = C_1 = 0$. The remaining boundary conditions (free end) take the form:

$$\sum_{n=0}^{\infty} C_n n(n-1) = 0 \tag{23a}$$

$$\sum_{n=0}^{\infty} C_n n(n-1)(n-2) = 0 \tag{23b}$$

Now, the complex form of Equation (23) was transformed to the real-valued one; representing solution w as

$w = w_1 + iw_2$

(24)

and complex coefficients

$C_n = A_n + iB_n$

(25)

such that

$w_1 = \sum_{n=0}^{\infty} A_n x^n$

(26a)

$$w_2 = \sum_{n=0}^{\infty} B_n x^n$$

(27b)

where A_n and B_n are real constants, which can be obtained by substituting Equations (24) - (28) into Equation (31), producing the following recurrent equations for the coefficients A_n and B_n

$$A_n = \frac{1}{n(n-1)(n-2)(n-3)}\{-\varepsilon\beta A_{n-2}(n-2)(n-3) \\ -2\varepsilon\eta A_{n-3}(n-3) + (\omega^2 - \eta^2)A_{n-4} \\ +2\varepsilon\omega B_{n-3}(n-3) + 2\eta\omega B_{n-4}\}$$

(28a)

$$B_n = \frac{1}{n(n-1)(n-2)(n-3)}\{-\varepsilon\beta B_{n-2}(n-2)(n-3) \\ -2\varepsilon\eta B_{n-3}(n-3) + (\omega^2 - \eta^2)B_{n-4} \\ -2\varepsilon\omega A_{n-3}(n-3) - 2\eta\omega A_{n-4}\}$$

(28b)

With these equations, the boundary conditions (25a) and (25b) can be rewritten as:

$aC_2 + bC_3 = 0,$

(29)

$cC_2 + dC_3 = 0,$

where *a*, *b*, *c*, and *d* are complex constants. Specifying, for example $C_1 = 1$ ($A_1 = 1$ and $B_2 = 0$), and $C_3 = 0$ ($A_3 = B_3 = 0$), from equations (25a) and (25b) one can have:

$$a = \sum_{n=0}^{\infty} A_n n(n-1) + iB_n n(n-1)$$

(30)

$$c = \sum_{n=0}^{\infty} A_n n(n-1)(n-2) + iB_n n(n-1)(n-2)$$

Similar to the previous case, if one specifies $C_2 = 0$ and $C_3 = 1$, i.e. one sets $A_2 = B_2 = B_3 = 0$, $A_3 = 1$, then one could obtain

$$b = \sum_{n=0}^{\infty} A_n n(n-1) + iB_n n(n-1)$$

(31)

$$d = \sum_{n=0}^{\infty} A_n n(n-1)(n-2) \\ + iB_n n(n-1)(n-2)$$

where A_n and B_n are given by recurrent equations (29a) and (29b).

The condition of existence of a non-trivial solution for the homogeneous system of linear algebraic equations (30) is that the determinant formed from the coefficients must be equal to zero or

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = 0$$

(32)

from which the critical parameters of the initiation of the dynamic instability can be obtained by means of a theoretical procedure for determining the roots of a non-linear algebraic equation.

4.0 DISCUSSION

Results of the theoretical calculations of the critical parameters corresponding to the initiation of dynamic instability, simply supported pipes with flowing water are given in Fig.2. From this figure, the area below the curves corresponded to the case when the flowing medium generated the damping effect, and in the area above the theoretical curves, it powered the development of unstable and unbound deflections.

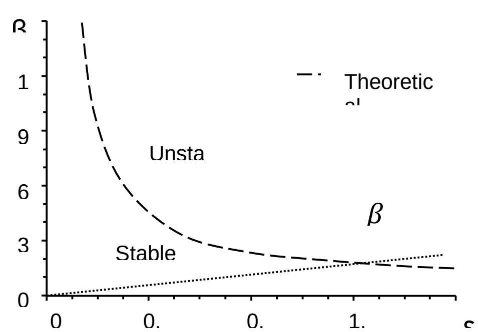


Fig. 2: Critical conditions for initiation of dynamic instability

This response was governed by two non-dimensional parameters incorporating mechanical properties of the pipe and the characteristics of the flowing medium. As an example of the usefulness of the obtained results, the initiation of dynamic instability investigated in this work had been considered as a major risk factor for high pressure aboveground pipelines as a full bore failure can provoke the development of such behavior. In this case, a significant damage can be expected due to chaotic movements of the pipe.

The derived conditions of the initiation of instability were considered as a pretty conservative estimate as the present mathematical model did not take into account other important factors, for example, friction or gravity, which could contribute significantly to the damping properties and the energy absorption mechanisms.

To assess the relevance of the derived conditions of the initiation of instability to practical problems, consider, for example, a full bore failure in a hypothetical aboveground gas flowline of a diameter D_i = 160 mm and wall thickness t = 4 mm, subjected to operating pressure P_o = 10 MPa. Let the gas density at operating conditions be $\square_o$ = 100 kg/m^3 and the specific heat ratio of the gas, $\square$ = 1.3, which represent typical values for raw natural gas. The velocity of the gas flow through the orifice can be roughly estimated using the simplified gas dynamics model presented [17] and this was found to be around 350 m/s. The gas flow density behind the decompression wave was approximately 60 kg/m^3. Furthermore, one can determine the ratio $\beta/\varepsilon = \frac{\lambda_p+\lambda_f}{\lambda_f} \approx 15$. The line $\beta = 15\varepsilon$ intersects the critical curve (see Figure 10) at $\varepsilon = 1.25$. From equation (8b), the critical length of the pipe was found to be around 15 meters.

This example demonstrates that the pipe whip phenomenon or dynamic instability is relevant to the typical operating conditions that normally occur in high-pressure aboveground pipelines of relatively small diameter; and to avoid this potentially dangerous dynamic effect, the pipe has to be restrained against lateral movements every 15 meters. The anchor spacing can be increased by selecting pipes with large moment of inertia (lager diameter and wall thickness) and with the reduction of the operating pressure or gas density.

The equations derived here may also be useful in reducing potential damage from other equipment working under high-pressure conditions. The criterion (Figure 10) can be applied, for example, when designing pipe attachments, exhausts or dispensers of relatively small diameter or flexible connections.

5.0 CONCLUSION

In this study, an analytical perturbation technique was developed to analyze the transient response of a simply supported pipe with flowing water subjected to dynamic loading. This response was governed by two non-dimensional parameters incorporating the mechanical properties of the pipe and the characteristics of the flowing medium. Besides, the conditions of the initiation of the dynamic instability in the case of a simply supported type of the pipe support conditions were investigated. Nevertheless, a similar approach could be adopted for investigation of transient response of a pipe with other support and loading conditions, as well as for the determination of the conditions of the initiation of dynamic instability. For example, as for the usefulness of the obtained results, the initiation of dynamic instability investigated in this work had been considered as a major risk factor for high pressure aboveground pipelines as a full bore failure can provoke the development of such behavior. In this case, a significant damage can be expected due to chaotic movements of the pipe. The derived conditions of the initiation of instability could be considered as a pretty conservative estimate as the present mathematical model did not take into account other important factors, for example, friction or gravity, which could contribute significantly into the damping properties and energy absorption mechanisms. The derived model represents the simplest extension of the Bernoulli-Euler beam theory, and, of course, could not capture all mechanisms and effects associated with pipe-flow interactions, and thus, more sophisticated models might be required for more accurate assessments. The latter will be a subject for further investigations.

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